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RESEARCH ARTICLE

Informational leverage: the problem of noise traders

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Abstract The literature suggests that security design can be used to manipulate the information content of securities prices [what is referred to as the "informational leverage effect" in Boot and Thakor (J Finance **48**, 1349–1378, 1993)]. The informational leverage effect arises in this literature in a market microstructure environment in which noise trade is exogenous, which is a fairly standard assumption dating back to the framework developed in Grossman and Stiglitz (Am Econ Rev **70**, 393–408, 1980). This assumption is relaxed in our paper, and we show that the informational effects described in the related literature become less clear cut when noise trading activity is endogenous. We find that the intensity and direction of these effects depends crucially on the parameters describing the modeling environment. The elegant point of the informational leverage literature is that these effects arise largely independently of such parameters, but with endogenous noise trading that is no longer true. This literature may, therefore, lead to too strong conclusions being drawn about the relationship between information revelation and security design.

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1 Introduction

An idea put forward in the security design literature is that the informational content of securities' prices can be controlled by the issuer, the so called "informational leverage effect". Security design can, therefore, be used as a device to extract private information from informed traders. By influencing the sensitivity of a security to new information, the issuer is indirectly influencing the information gathering process of speculators. Thus, the social information gathering process is indirectly controlled by security design which can have important effects. For instance, the decision making process can be improved with more information revelation in asset prices; risk sharing benefits can be protected by discouraging informed speculation; or issuers who benefit from the proceeds of selling security can manipulate the information gathering process to increase the value of undervalued securities or protect the value of overvalued securities. Sometimes these effects interact, as in Boot and Thakor (1993) where "good" firms issue maximally information sensitive equity to protect against selling undervalued stock and "bad" firms must follow to avoid direct revelation of information. Another example is Fulghieri and Lukin (2001) where the firm seeks to maximize the probability of a successful issue, and this is done by issuing maximally information sensitive equity which encourages information gathering. Both papers argue the firm can (and will) control the information content of the prices of its securities by altering their capital structure. The literature assumes a response to capital structure changes is made by the informed traders, whereas a corresponding response is not made by the so called noise traders as their behavior is either considered irrational or driven by factors outside the model. This is clearly a shortcoming of the literature. This paper seeks, therefore, to analyze the actions of noise traders in response to attempts at manipulating the informational leverage of assets.

We analyze, therefore, the capital structure choice of a firm in a model in which all investors' demand for securities is endogenously determined. There is obviously a question of what is brought into the framework when noise traders' behavior is endogenized in this way. The idea of "irrational" noise trade is not necessarily that they represent irrational behavior per se but rather that they represent behavior that is not modeled explicitly within the framework. Endogenizing this behavior is of course problematic if we are not endogenizing the correct, previously not modeled, underlying behavior. To make this choice as uncontroversial as possible we assume the simplest of rationalizations, namely that noise traders are risk averse traders who trade due to exogenous (noisy) shocks to their endowment risk. In effect, we transfer the exogenous noisy random variable from the point of demand to the point of endowment risk which triggers hedge motivated demand.

In our model, a firm issues debt and equity to finance a risky investment opportunity. The debt contract is held by a risk neutral bank and the equity is traded in a Kyle-type microstructure market. There are two types of investors participating in the equity market. The first, the hedgers, trade because they are risk-averse and seek to hedge an income risk. The second, the speculators, trade because they have invested in an information signal about the future value of the investment the firm undertakes. There is also a competitive risk neutral market maker whose role is to clear the market by setting the appropriate price. Both Boot and Thakor (1993) and Fulghieri and Lukin (2001) have setups which are similar to that described here, and they find that the capital structure decision impacts on the information revelation of equity prices and the probability that the proceeds from the issues exceed a given level, i.e., that the capital structure decision causes an informational leverage effect. The essential difference between these studies and ours is that we employ rational, risk averse hedgers as a source of noise trade.

In the main body of the paper we develop a framework with endogenous noise trade in which the capital structure decision will not change the distributional properties of the firm values in the secondary market at all. In this setting, therefore, the firm cannot take advantage of good information by changing its capital structure to attract informed speculators (as in Boot and Thakor 1993) and it cannot affect its value at risk when issuing new capital (as in Fulghieri and Lukin 2001). We discuss some generalizations of the model towards the end, in which informational effects emerge but we show in this case that the effects become critically dependent on exogenous parameters describing the modeling environment. The main message in our paper is, therefore, that non-strategic noise trading is not an innocent assumption in this strand of work and has bearing on the results. Drawing conclusions about the information revelation process of market prices is difficult, therefore, without reference to the modeling framework.

The immediately related literature is the one using informational leverage in a security design context. Boot and Thakor (1993) show that issuing multi-layer securities encourages informed speculation when the firm has superior information over outside investors. This informational leverage increases the market value of high-earning firm and will thus be chosen in equilibrium. Fulghieri and Lukin (2001) explore a security design problem when the firm has superior information about its future earnings over outside investors and the investors costly acquire the information. They find that issuing debt is optimal when the cost of information acquisition is high while issuing equity is optimal when the cost is low. Critically, both papers assume that a part of the total demand for securities is exogenously given, which is the essential point at which our paper differs. Other papers have, as we do, measured welfare effects in a Kyle framework, for instance Dow (1998) and Dow and Gorton (1997), but these papers do not look at informational leverage effects directly. Our model is however innovative in relation to the latter strand in the sense that it combines the study of the welfare effects of capital structure in a Kyle framework in which (noisy) hedge demand has continuous support. In Dow (1998) and Dow and Gorton (1997) the Kyle framework is stripped down to a setting with binary (noisy) hedge demand, which is relaxed here. Our model is also related to a strand of literature which analyzes links between security design and information revelation in noisy rational expectations markets, see Marin and Rahi (1999, 2000). We draw several parallels to this literature towards the end of the paper.

The remainder of this paper is organized as follows. Section 2 outlines the model. Section 3 derives the equilibrium in a benchmark case with exogenous noise trade. Section 4 derives the equilibrium with endogenous noise trade. Section 5 discusses some generalizations of the model. Section 6 concludes.

2 The model

The model consists of three periods; an initial period in which the capital structure decision is made (ex ante), an intermediate period in which the the issued securities are traded (trading period), and a final period in which the firm is liquidated (liquidation period). Ex ante, the firm is endowed with a risky project which costs $v \geq 1$ to start, and generates stochastic earnings $\tilde{V}$ in the liquidation period which are either $v + 1$ or $v - 1$, with equal probabilities. The realization value of the earnings is known privately to the firm but not to the general market. The firm finances the investment by issuing bank debt with a face value $F \leq v + 1$, and residual equity. The firm may of course also issue traded debt but in our model traded debt has trivial impact on the results and can be regarded as equity as long as it is junior to the bank debt. This point will be discussed later.

The bank who owns the non-trivial bank debt does not learn the true debt value until the liquidation period, is risk neutral and behaves competitively. Thus the bank purchases such debt at its unconditional liquidation value, which is

$$D(F) = \frac{1}{2}F + \frac{1}{2}\min(F, v - 1) = F + \frac{1}{2}\min(0, v - 1 - F) \tag{1}$$

The debt is of course risky if $F > v - 1$.

The residual equity (or traded junior debt plus equity) is traded in the stock market which is cleared by a risk neutral market maker who clears the order flow sequentially in a market with a continuum of traders. The payoff of the equity is $\tilde{Y}$ which is given by

$$\tilde{Y} = \max(\tilde{V} - F, 0) \tag{2}$$

There are two types of investors participating in the equity market – hedgers and speculators. The hedgers are risk averse and their utility function is a simple piecewise linear function

$$u = \min(\gamma W, W) \tag{3}$$

where $\gamma \in (0, 1)$ and W denotes wealth, which may be positive or negative. The utility function has a kink at zero wealth, with wealth losses given a weight of 1 and wealth gains a weight of $\gamma < 1$. The hedgers become more risk averse, therefore, as the value of γ approaches zero. We assume all hedgers have the same risk aversion coefficient γ. The hedgers are infinitesimal with a total mass of unity. The hedgers can trade the equity to hedge their income risk. There are two types of income risk. The first type is perfectly negatively correlated with the asset value innovation. This means that the terminal income is $+1$ if the terminal asset value is $v - 1$ and -1 if the terminal asset value is $v + 1$. The hedgers with this type of income risk can, therefore, hedge their position by buying the equity. The other type is perfectly positively correlated, i.e., the terminal income is $+1$ if the terminal asset value is $v + 1$ and -1 if the terminal asset value is $v - 1$. The hedgers with this type of income risk can, therefore, hedge their position by selling the equity. We assume the measure of hedgers with the first type of income risk is β and the measure of hedgers with the second type of income risk is $1 - \beta$. We assume that β is a random draw from a uniformly distributed random variable, $\beta \sim U[\frac{1}{2} - \frac{\xi}{2}, \frac{1}{2} + \frac{\xi}{2}]$. The parameter $\xi \in [0, 1]$ is known to all parties, but the realization of the random draw of β is never revealed to any agent in the model.

The speculators are risk neutral and have the opportunity of buying a perfect signal of the future asset value. The information cost is a constant dead weight cost of c. The speculators are also infinitesimal. We assume the measure of speculators available to buy the information signal is large so that there are always agents free to acquire information if this is profitable. The measure of speculators that actually invest in the information signal is α.

We now describe market clearing which is a version of Kyle's (1985) one-shot game. The market maker observes all incoming orders from the speculators and the hedgers before setting prices. Let x denote the order from each individual speculator, and y_1 and y_2 the order from each individual hedger of the first and second type, respectively. The aggregate demand, once all orders have arrived, is

$$\tilde{Q} = \alpha x + \tilde{\beta} y_1 + (1 - \tilde{\beta}) y_2 \tag{4}$$

We allow the market maker to observe individual orders. The market maker is able to work out the equilibrium trading quantities of the hedgers, as he has as much information as they have. The market maker can, therefore, infer the true asset payoff from observing a single order x whenever $x \neq y_1, y_2$, i.e., when a speculator trades in quantities that differ from the quantities that would be traded by a corresponding hedger on the same side of the market. In equilibrium, therefore, the speculators trade in quantities similar to those of the hedger on the same side of the market as otherwise prices become fully revealing and no speculative profits can be made. The market maker cannot observe the realization of β but is aware of its distribution, and can, moreover, infer the value of α along the equilibrium path. The market price is, therefore, an informationally efficient price that is determined after all orders have arrived:

$$P = E(\tilde{Y} | Q, \alpha, \xi, \text{individual orders}) \tag{5}$$

In equilibrium, the market maker cannot gain extra information from observing individual orders, so in effect the equilibrium price becomes a Kyle (1985) price that depends on the aggregate order flow only.

We operate with two firm objectives. First, a firm that has already financed the investment cost v seeks to adopt a capital structure that maximizes the expected firm value conditional on the true value of its assets V. This is, in effect, the objective set in Boot and Thakor (1993). Second, a firm that has yet to finance the investment cost v

seeks to adopt a capital structure that maximizes the likelihood of financing the project, also conditional on the true value V. If the proceeds from selling debt and equity exceeds the financing cost v, the firm retains equity as a surplus to its original owner. This is the objective set in Fulghieri and Lukin (2001). Both objectives recognize the firm as a strategic, informed agent whose only scope of action is the capital structure decision.

We assume away all frictions (taxes, bankruptcy costs, and issuing costs) and also that the discount rate is zero. All non-trivial effects are, therefore, driven by information heterogeneity among the investors in the equity market.

The model aims to investigate two things. The first is the link between the informativeness of the prices formed in the trading period and the capital structure decision made ex ante as in Boot and Thakor (1993). The second is the risk of failing to raise sufficient funds to finance an investment project as in Fulghieri and Lukin (2001). We use the same market structure, therefore, as both a primary market and a secondary market. The chosen structure resembles a secondary market (a market maker, or specialist, market such as the New York Stock Exchange) but not really an IPO market. In practice one is not permitted to submit both buy and sell orders in an IPO market [although, incidentally, this market structure is used to model the IPO market in Fulghieri and Lukin (2001)]. There are two ways in which this could be corrected. One is to assume there is a book-building process and a formal auction going on at a date before the equity is traded, and that this process generates a signal of the aggregate order flow in the secondary market. If this signal indicates a sufficiently low secondary market price the investment is pulled. Another way is simply to ignore all sell orders and work through the model as a pure no short selling Kyle market, something for instance Boot and Thakor (1993) effectively implement for informed traders. Both of these avenues represent valid modifications and extensions of our model but do in our view not shed additional light on the basic focus for the analysis.

As mentioned it is essential the debt is not tradable. A traded debt contract could always be used to synthesize the payoff of unlevered equity, making the firm's capital structure decision irrelevant by default. Barriers to forming synthetic unlevered equity is, therefore, necessary in order to generate non trivial results. In some ways this is also assumed indirectly in the related literature. For instance, in Boot and Thakor (1993) it is assumed that noise trade is exogenous—so noise traders will not seek to synthesize unlevered equity by assumption—making the capital structure decision a determinant for information revelation in the equity market.

The assumption that the market maker observes individual orders generates a natural way of modeling competition between informed traders. A mechanism for preventing perfect competition ex post is needed in order to support information gathering ex ante. Such mechanisms have also been used in the literature, for instance Boot and Thakor (1993) assume competition between informed traders is prevented through budget and short selling constraints.

3 Benchmark case—exogenous noise trading

In this section we derive the equilibrium of the model assuming exogenous noise trading, which enables us to link the model with the existing literature, in particular

Boot and Thakor (1993) and Fulghieri and Lukin (2001). Suppose the noise traders trade a fixed amount $y > 0$, where type 1 traders buy y units and type 2 traders sell y units of the asset. Also suppose initially there are no speculators present, i.e., $\alpha = 0$. The aggregate trading quantity is

$$\tilde{Q} = \tilde{\beta} y - (1 - \tilde{\beta}) y$$

which by the assumption outlined above is distributed uniformly on $[-\xi y, \xi y]$ with density function

$$f(Q) = \frac{1}{2\xi y} \quad \text{for } Q \in [-\xi y, \xi y]$$

Now suppose there are speculators present, i.e., $\alpha > 0$. If the speculators buy, they trade an aggregate (non-stochastic) quantity αy (each trader must trade in the same quantities as the noise traders in order to avoid making the market price fully revealing straight away). Hence, the aggregate trading quantity is uniform on $[-\xi y + \alpha y, \xi y + \alpha y]$, which has a density function expressed in terms of the one given above: $f(Q - \alpha y)$. Similarly, if the speculators sell, the density function can be expressed as $f(Q + \alpha y)$. The market maker's beliefs about the true asset value is, therefore, given by the following Bayesian revision process:

$$\begin{aligned}
\Pr(\text{Asset value is high} | Q, \alpha) &= \frac{f(Q - \alpha y)}{f(Q - \alpha y) + f(Q + \alpha y)} \\
&= \begin{cases} 1 & \text{if } Q > \xi y - \alpha y \\ \frac{1}{2} & \text{if } -\xi y + \alpha y \leq Q \leq \xi y - \alpha y \\ 0 & \text{if } Q > -\xi y + \alpha y \end{cases}
\end{aligned}$$

The price is, therefore, either fully revealing or not revealing at all. The speculators' profit function can be worked out on the basis of this revision process. Suppose the speculators buy the asset on the basis of positive information. Then the trading quantity is uniform on $[-\xi y + \alpha y, \xi y + \alpha y]$, and the price will be fully revealing if $Q > \xi y - \alpha y$. Therefore, the speculators make profits only if $Q \leq \xi y - \alpha y$, which happens with the following probability

$$\Pr(Q \leq \xi y - \alpha y) = \frac{(\xi y - \alpha y) - (-\xi y + \alpha y)}{(\xi y + \alpha y) - (-\xi y + \alpha y)} = 1 - \frac{\alpha}{\xi}$$

Here we must assume that $\alpha < \xi$, which will be shown to hold when α is eventually determined. The model is symmetric, so the speculators make profits with the same probability $1 - \frac{\alpha}{\xi}$ if they sell. Hence, the expected profit by the speculators is

Speculator's ex ante profit

$$= \frac{1}{2}\left(1-\frac{\alpha}{\xi}\right)\left(\left((v+1-F)-\frac{1}{2}((v+1-F)+\max(0,v-1-F))\right)\right.$$
$$\left.+\left(\frac{1}{2}((v+1-F)+\max(0,v-1-F))-\max(0,v-1-F)\right)\right)y$$
$$= \left(1-\frac{\alpha}{\xi}\right)\frac{1}{2}((v+1-F)-\max(0,v-1-F))y$$

Define the number $R(F)$ by

$$R(F)=\frac{1}{2}((v+1-F)-\max(0,v-1-F))$$

It is easy to see that this number is constant and equal to 1 for $F \leq v-1$ and decreasing linearly in F for larger values of F. This value can be thought of as a measure of the information sensitivity of the firm's equity. The speculators are likely to incur the information cost c to the point at which their expected trading profits equal the information cost, so the endogenously given measure of informed speculators is given by the equation

$$c=\left(1-\frac{\alpha}{\xi}\right)yR(F)$$

or, equivalently

$$\alpha=\xi\left(1-\frac{c}{yR(F)}\right)$$

Both Boot and Thakor's (1993) result as well as Fulghieri and Lukin's (2001) result are now transparent. A firm knowing its own type to be good (a firm whose true value is $v+1$) is maximizing its expected market value by maximizing the probability of information revelation $\frac{\alpha}{\xi}$, and therefore also maximizing the measure of informed speculators. This is done by maximizing the informational leverage $R(F)$, i.e., making the equity maximally information sensitive. Similarly, a firm knowing its own type to be good is maximizing (weakly) its value at risk at any level by maximizing the probability of information revelation, which is also done by maximizing α. Again, this is achieved by maximizing the information sensitivity of equity, i.e., making $R(F)$ maximal.

4 The equilibrium—endogenous noise trading

In this section we seek to establish the equilibrium. At the second stage in the model, the traders meet in the secondary market, and must submit optimal orders conditional on their type (i.e., conditional on being a hedger type 1, hedger type 2, or a speculator). At the first stage, the information gathering strategy for the risk neutral speculators must be rational in the sense that they expect to offset the information cost ex ante with expected trading profits ex post. Thus, the parameter α can be determined endogenously given what happens in the secondary market. The final wealth is given by

$$\tilde{w}_1(y_1) = y_1(\tilde{Y} - \tilde{P}) - (\tilde{V} - v) \tag{6.1}$$

for hedgers of the first type and

$$\tilde{w}_2(y_2) = y_2(\tilde{Y} - \tilde{P}) + (\tilde{V} - v) \tag{6.2}$$

for hedgers of the second type. In equilibrium, therefore, the hedgers submit orders

$$y_1 \in \arg\max_y E(u(\tilde{w}_1(y))) \tag{7.1}$$

and

$$y_2 \in \arg\max_y E(u(\tilde{w}_2(y))) \tag{7.2}$$

The speculators submit orders also, and their orders are optimally designed to by non-revealing at the individual level. Therefore, the equilibrium condition determining x is that

$$x \in \{y_1, y_2\} \tag{8}$$

where $x > 0$ if the information is positive ($\tilde{V} = v + 1$) and $x < 0$ if the information is negative ($\tilde{V} = v - 1$). Finally, the market maker determines an informationally efficient price

$$P = E(\tilde{Y}|Q, \alpha, \xi, \text{individual orders}) \tag{9}$$

Below we establish an equilibrium in which $x \in \{y_1, y_2\}$ and $y_1 = -y_2 = y$. Suppose α be given at this stage. Then we find the following.

Lemma 1 *There exists an equilibrium where the informed traders make positive profits if $\alpha < \xi$. In this equilibrium, the market maker sets equity prices as a function of the aggregate trading quantity Q. The prices are given by*

$$P(Q) = \begin{cases} v + 1 - F \\ \quad \textit{if } Q > \frac{2(\xi - \alpha)}{(v+1-F) - \max(0, v-1-F)} \\ \frac{1}{2}(v + 1 - F) + \frac{1}{2}\max(0, v - 1 - F) \\ \quad \textit{if } -\frac{2(\xi - \alpha)}{(v+1-F) - \max(0, v-1-F)} \leq Q \leq \frac{2(\xi - \alpha)}{(v+1-F) - \max(0, v-1-F)} \\ \max(0, v - 1 - F) \\ \quad \textit{if } Q < -\frac{2(\xi - \alpha)}{(v+1-F) - \max(0, v-1-F)} \end{cases} \tag{10}$$

The hedgers trade quantities

$$|y_1| = |y_2| = \frac{2}{(v + 1 - F) - \max(0, v - 1 - F)} \tag{11}$$

where $y_1 > 0 > y_2$*, and finally the speculators trade quantities* $|x| = |y_1| = |y_2|$*. The speculators buy if the information is positive and sell if it is negative.*

The proof can be found in the Appendix. The assumption that the aggregate difference between the measure of the two types of hedgers is uniformly distributed has two modeling advantages. First, it makes the optimal price setting mechanism very simple. The prices are either fully revealing or not revealing at all. This is a property that is shared by the models employing bivariate aggregate noise trade (Dow 1998; Dow and Gorton 1997 are examples). Second, because the price setting is relatively simple, it becomes very easy to work out the expected utility from hedging. This facilitates the derivation of welfare statements for the uninformed traders.

All traders' actions are observable to the market maker although the market maker cannot distinguish a speculator and a hedger by means other than observing the trader's actions. This is a constraint that has greater bearing on the strategy of the informed traders than the strategy of the uninformed. The informed traders must, in effect, constrain their order submission strategy to orders that are observationally close to those submitted by uninformed traders on the same side of the market. A major modeling advantage is that the informed traders are prevented from competing too aggressively against each other. Therefore, they can still expect to make trading gains even after the information cost is sunk. We use this in the following to determine the equilibrium measure of informed traders, α^*.

Lemma 2 *Suppose* $1 \geq \xi \geq c$ *(the first inequality is assumed at the outset). In equilibrium, there are* α^* *active speculators who invest in information, where*

$$\alpha^* = 2\xi(1 - c) \tag{12}$$

The probability the price is fully revealing is equal to $1 - c$*.*

The proof is in the appendix. We notice from Lemma 2 that the capital structure decision, F, does not enter into the decision parameters to the informed speculators. There is, consequently, no informational leverage effect arising in equilibrium.

Proposition 1 *The capital structure decision has no impact on the distributional properties of the future firm value or on the welfare of the agents who trade the firm's equity.*

It may be useful at this point to recall what goes on in the related literature. In Boot and Thakor (1993), the essential effect is driven by the fact that a "good" firm seeks to send an information signal to the market about its true intrinsic value as this is likely to increase its expected value (conditional on the signal). There is no direct communication channel available, but the firm can issue multiple securities to attract informed speculators to several markets. If one security is more sensitive to the firm's information than the other one, the market for this security is likely to attract more informed trade than the other market and consequently reveal more information. Boot and Thakor (1993) outline a set of conditions under which the "aggregate" information revelation process is non-linear in the capital structure decision, and work out the optimal capital structure as a result.

We show that the non-linearity between capital structure and aggregate information revelation breaks down when we assume that noise traders are utility maximizers seeking to hedge an endowment risk, and hence react to the capital structure decision of the firm. They react, moreover, in such as way that there are no more informed traders arriving when the equity is more risky as when it is less risky. Why is this? The answer is surprisingly simple. The uninformed traders need to trade more when the security is less information sensitive to achieve the desired hedge effect. The volume of "noise trade" does, therefore, increase. The informed traders can in return also trade in larger volume when the security is less information sensitive, but the profit potential is smaller since the prices move less. The net effect is the product of the quantity effect (which is positive in this case) and the price effect (which is negative) and is exactly zero since the quantity effect is directly inversely related to the price effect through the hedgers' utility maximization program. The product of the two, therefore, remains independent of the capital structure decision. This explains why Fulghieri and Lukin's (2001) result does not survive our modification either, as the value at risk of firm value is unaffected by the capital structure.

We find that the capital structure decision is welfare neutral also, despite the fact that the traders are restricted to trading levered equity only (the debt is non-tradable). This is fairly obvious when we talk about the informed speculators. The speculators pay an information cost ex ante to earn trading profits ex post. When the market for information is perfectly competitive, they earn zero net expected profits overall. The welfare effect is somewhat less trivial for the uninformed hedgers, as the hedgers' trading activity is a function of the capital structure decision (see Lemma 1). The hedgers' expected utility from trading is given by the following expression (derived in the appendix in the proof of Lemma 1—the expression shows the expected utility for a type 1 hedger but by the symmetry of the model we get a similar expression for a type 2 hedger):

$$E[u(\tilde{w}_1(y))]$$
$$= -\frac{1}{2}\left(\int_{-\xi y+\alpha y}^{\xi y-\alpha y}\left(y\frac{(v+1-F)-\max(0, v-1-F)}{2}-1\right)\frac{1}{2\xi y}dQ+\int_{-\xi y-\alpha y}^{-\xi y+\alpha y}\frac{1}{2\xi y}dQ\right)$$

$$+\frac{\gamma}{2}\left(\int\limits_{-\xi y+\alpha y}^{\xi y-\alpha y}\left(y\frac{(v+1-F)-\max(0,v-1-F)}{2}+1\right)\frac{1}{2\xi y}dQ+\int\limits_{\xi y-\alpha y}^{\xi y+\alpha y}\frac{1}{2\xi y}dQ\right) \tag{13}$$

Substituting the optimal hedge demand y^* into this expression, we find an expression in α^* which can be identified:

$$E[u(\tilde{w}_1(y^*))]=-\frac{(1-\gamma)\alpha^*}{2\xi}=-(1-\gamma)(1-c) \tag{14}$$

and which does not depend on the capital structure decision at all. Therefore, the hedgers' expected utility is also neutral with respect to the capital structure decision of the firm. This implies irrelevance of capital structure in a market microstructure environment, contrary to what is asserted in the informational leverage literature.

5 Generalizations

How robust is the result in Proposition 1? After all, we are employing a fairly specific model both in terms of preferences (piecewise linear utility functions and endowment risk that is perfectly correlated with unlevered equity) and distributions (uniformly distributed groups of noise traders). Following the discussion in the previous section we discuss in this section the robustness of Proposition 1. Much of this discussion essentially boils down to whether we can write the hedgers' optimal demand by Eq. (11) given in Lemma 1, and this will be the focus for the first part of this section. As long as the market does not allow for partial information revelation (prices are either fully informative or not informative at all—see Lemma 1), Eq. (11) *implies* that capital structure effects are irrelevant. Therefore, we are first searching for generalizations which may violate the optimality of Eq. (11). The second half of the discussion in this section centers around a generalized model which allows for partial information revelation.

We look first at the assumption regarding preferences. Suppose the hedgers have a utility function $u(\tilde{w}_i(y))$ (where $i=1$ for type 1 hedgers and $i=2$ for type 2 hedgers) which is strictly concave. The hedgers seek solutions to the problems $y_1 \in \arg\max_y E[u(\tilde{w}_1(y))]$ and $y_2 \in \arg\max_y E[u(\tilde{w}_2(y))]$, which is symmetrical around zero by the symmetry of the model. Hence, there is either full information revelation or no information revelation at all, with prices equal to $v+1-F$ for aggregate order flow $Q>\xi y-\alpha y$, $\max(0, v-1-F)$ for aggregate order flow $Q\leq -\xi y+\alpha y$, and finally $\frac{1}{2}((v+1-F)+\max(0,v-1-F))$ for $-\xi y+\alpha y<Q\leq \xi y-\alpha y$. The expected utility for the hedgers can, therefore, be expressed as the following. We derive the expression assuming a type 1 hedger, which is without loss of generality due to the symmetry of the model.

$$\begin{aligned}
&E[u(\tilde{w}_1(y))] \\
&= \frac{1}{2}\left(\int\limits_{-\xi y-\alpha y}^{-\xi y+\alpha y} \frac{u(-1)}{2\xi y} dQ\right. \\
&\qquad \left. + \int\limits_{-\xi y+\alpha y}^{\xi y-\alpha y} u\left(y\frac{(v+1-F)-\max(0,v-1,F)}{2}-1\right)\frac{1}{2\xi y}dQ\right) \\
&\quad + \frac{1}{2}\left(\int\limits_{-\xi y+\alpha y}^{\xi y-\alpha y} u\left(-y\frac{(v+1-F)-\max(0,v-1-F)}{2}+1\right)\frac{1}{2\xi y}dQ\right. \\
&\qquad \left. + \int\limits_{\xi y-\alpha y}^{\xi y+\alpha y} \frac{u(1)}{2\xi y} dQ\right)
\end{aligned} \tag{15}$$

Integrating out each term, we find

$$\begin{aligned}
E[u(\tilde{w}_1(y))] = {} & \frac{\alpha}{2\xi}(u(-1)+u(1)) \\
& + \frac{(\xi-\alpha)}{2\xi}\left(u\left(y\frac{(v+1-F)-\max(0,v-1-F)}{2}-1\right)\right. \\
& \left. + u\left(-y\frac{(v+1-F)-\max(0,v-1-F)}{2}+1\right)\right)
\end{aligned} \tag{16}$$

Assuming u is differentiable, the first order condition for expected utility maximization with respect to y is given by the following expression

$$\begin{aligned}
\frac{dE[u(\tilde{w}_1(y))]}{dy} = {} & \frac{(v+1-F)-\max(0,v-1-F)}{2} \\
& \times \left(u'\left(y\frac{(v+1-F)-\max(0,v-1-F)}{2}-1\right)\right. \\
& \left. - u'\left(-y\frac{(v+1-F)-\max(0,v-1-F)}{2}+1\right)\right) = 0
\end{aligned} \tag{17}$$

This condition is clearly satisfied for a risk averse agent only if the optimal hedge demand satisfies the condition in (11) given in Lemma 1. More general preferences will, therefore, not in itself change the conclusions in Proposition 1. These conclusions were to a large extent built on the fact that the hedgers' demand is inversely related to the price effects as is indeed the case when inverting (17) to derive the optimal hedge demand. The seemingly restrictive preferences in (3) do not, therefore, in isolation play a crucial role in underpinning Proposition 1.

Next, we look quickly at the assumptions regarding the endowment risk of the hedgers. Suppose the hedgers' final wealth is given by

$$\tilde{w}_1(y_1) = y_1(\tilde{Y} - \tilde{P}) - \theta(\tilde{V} - v) \tag{18.1}$$

$$\tilde{w}_2(y_2) = y_2(\tilde{Y} - \tilde{P}) + \theta(\tilde{V} - v) \tag{18.2}$$

which generalize the analysis in the previous section by the inclusion of the parameter $\theta \in (0, \infty)$. The special case of $\theta = 1$ yields Eqs. (6.1) and (6.2). Extending the analysis in the previous section in a straightforward way, we find that the first order conditions for utility maximization will be satisfied for

$$|y_1| = |y_2| = \frac{2\theta}{(v + 1 - F) - \max(0, v - 1 - F)} \tag{19}$$

which involves just a simple scaling of the hedgers' demand for equity. The hedgers' final wealth becomes, therefore, independent of the debt levels F as before, and all informational effects vanish. We summarize these findings in the following result.

Proposition 2 *Generalizing the model to arbitrary preferences u which are strictly increasing and concave, or to endowment risk which is an arbitrary multiple of the risk of unlevered equity will not change the conclusions in Proposition 1.*

What happens in the case with endowment risk which is not perfectly correlated with the asset payoff? We can look at a simple example with the piecewise linear utility function in (3), which is not too involved. Suppose the hedgers' wealth is written as

$$\tilde{w}_1(y_1) = y_1(\tilde{Y} - \tilde{P}) - \tilde{Z} \tag{20.1}$$

$$\tilde{w}_2(y_2) = y_2(\tilde{Y} - \tilde{P}) + \tilde{Z} \tag{20.2}$$

where $\tilde{Z}$ is a bivariate $+1/-1$ random variable with equally likely outcomes and

$$\Pr(Z = V - v) = \delta > \Pr(Z = -(V - v)) = 1 - \delta \tag{21}$$

Taking $\delta = 1$ we arrive back at our original model. Since we assume $\delta > 1 - \delta$ the random variable $\tilde{Z}$ and $\tilde{V} - v$ are positively correlated. Therefore, we expect a hedger with endowment risk $-\tilde{Z}$ is potentially interested in buying the asset to provide protection (which is imperfect) against the endowment risk. The question is how much he plans to buy, and in particular whether he chooses the quantity given in Eq. (11). To investigate this case we take α as given and work out the optimal trading quantity y as a function of α. If we find that y is independent of α we are done. Let us also by symmetry consider a hedger with endowment risk $-\tilde{Z}$ only, i.e., a hedger of type 1. From the previous analysis we can work out the probability of full information revelation in the market as $\frac{\alpha}{\xi}$ and the probability of no information revelation in the market as $\frac{\xi - \alpha}{\xi}$ (we know that $\xi > \alpha$ from the equilibrium condition on α). Conditioning on the random variables $\tilde{Z}$ and $\tilde{V} - v$ we can, therefore, work out the expected utility of the hedger as

$$\begin{aligned}
Eu(w_1) = &\frac{1}{2}\frac{\xi-\alpha}{\xi}\left(\delta\left(y\frac{(v+1-F)-\max(v-1-F,0)}{2}-1\right)\right.\\
&\left.+(1-\delta)\gamma\left(y\frac{(v+1-F)-\max(v-1-F,0)}{2}+1\right)\right)\\
&+\frac{1}{2}\frac{\alpha}{\xi}\left(-\delta+\gamma(1-\delta)\right)\\
&+\frac{1}{2}\frac{\xi-\alpha}{\xi}\left(-\delta\gamma\left(y\frac{(v+1-F)-\max(v-1-F,0)}{2}-1\right)\right.\\
&\left.-(1-\delta)\left(y\frac{(v+1-F)-\max(v-1-F,0)}{2}+1\right)\right)\\
&+\frac{1}{2}\frac{\alpha}{\xi}\left(\delta\gamma-(1-\delta)\right)\\
=&-\frac{1-\gamma}{2}+\frac{1-\gamma}{2}\frac{\xi-\alpha}{\xi}(2\delta-1)y\frac{(v+1-F)-\max(v-1-F,0)}{2}
\end{aligned} \tag{22}$$

We have here made the natural assumption that $y \leq \frac{2}{(v+1-F)-\max(v-1-F,0)}$ but this is not essential as we would arrive at the same conclusion making the opposite assumption. We find, taking the upper bound on y into account, that the hedger's welfare is maximized for y maximal, i.e., $y = \frac{2}{(v+1-F)-\max(v-1-F,0)}$, which is indeed both independent of α and identical to the expression given in equation (11). This suggests extending Proposition 1 to the case of imperfect correlation, however, this conclusion is problematic. It turns out that the piecewise linear form in (3) is essential in order to arrive at (22). Therefore, we operate in effect with a special case as far as preferences go and Proposition 1 cannot be generalized to the case of imperfect correlation between endowment risk and asset payoffs with arbitrary preferences. In general, the first order conditions for expected utility maximization with a function that is concave everywhere would not be satisfied at (11) and would generate a trading quantity y that is strictly less than $\frac{2}{(v+1-F)-\max(v-1-F,0)}$ for $\delta < 1$.

Finally, we look at the distribution of the parameter β that denotes the relative difference between the measure of type 1 and type 2 hedgers. We made earlier the computationally convenient assumption that β is uniformly distributed, and we will discuss below an alternative distributional assumption. This turns out to be a much more involved change of the model than those discussed above and needs to be analyzed in finer detail. The main reason for this is that market prices now in general become partially revealing. Suppose the measures of type 1 and type 2 traders are given by β_1 and β_2 respectively, and assume that the difference between them, i.e., $\tilde{\beta}_1 - \tilde{\beta}_2$, is normally distributed:

$$(\tilde{\beta}_1 - \tilde{\beta}_2) \sim N(0, \sigma^2) \tag{23}$$

In a symmetric framework, the hedgers who buy and sell submit the same quantity when they demand stock so the distribution of noisy demand is also normal

$$(\tilde{\beta}_1|y| - \tilde{\beta}_2|y|) \sim N(0, |y|^2\sigma^2) \tag{24}$$

The measure of informed traders is as before α. These traders will also as before behave as hedgers except they will always either just buy or just sell. The informed trade is, therefore, in aggregate the quantity $\alpha|y|$. Consequently, the density function describing aggregate demand is $f(Q - \alpha|y|)$ or $f(Q + \alpha|y|)$ depending on whether the informed traders buy or sell, respectively, where

$$f(Q) = \frac{1}{\sqrt{2\pi}|y|\sigma} \exp\left\{-\frac{1}{2}\left(\frac{Q}{|y|\sigma}\right)^2\right\} \tag{25}$$

When the market maker observes an aggregate order flow Q, therefore, he knows that this is drawn from the distribution $f(Q + \alpha|y|)$ with probability $\frac{1}{2}$ and the distribution $f(Q - \alpha|y|)$ with probability $\frac{1}{2}$. His revised beliefs are, therefore, given by the conditional probabilities

$$\begin{aligned}\Pr(\text{Low State}|Q) &= \frac{f(Q + \alpha|y|)}{f(Q + \alpha|y|) + f(Q - \alpha|y|)} \\ &= \frac{\exp\left(-\frac{\alpha}{|y|}\frac{Q}{\sigma^2}\right)}{\exp\left(\frac{\alpha}{|y|}\frac{Q}{\sigma^2}\right) + \exp\left(-\frac{\alpha}{|y|}\frac{Q}{\sigma^2}\right)}\end{aligned} \tag{26.1}$$

and

$$\begin{aligned}\Pr(\text{High State}|Q) &= \frac{f(Q - \alpha|y|)}{f(Q + \alpha|y|) + f(Q - \alpha|y|)} \\ &= \frac{\exp\left(\frac{\alpha}{|y|}\frac{Q}{\sigma^2}\right)}{\exp\left(\frac{\alpha}{|y|}\frac{Q}{\sigma^2}\right) + \exp\left(-\frac{\alpha}{|y|}\frac{Q}{\sigma^2}\right)}\end{aligned} \tag{26.2}$$

These conditional probabilities have the natural property that the beliefs remain unchanged when the market maker observes the neutral aggregate demand $Q = 0$, and that $\Pr(\text{Low State}|Q) < \frac{1}{2}$ for $Q > 0$ and $\Pr(\text{Low State}|Q) > \frac{1}{2}$ for $Q < 0$. The probabilities give us the price projection conditional on Q:

$$P(Q) = \frac{(v + 1 - F)\exp\left(\frac{\alpha}{|y|}\frac{Q}{\sigma^2}\right) + \max(0, v - 1 - F)\exp\left(-\frac{\alpha}{|y|}\frac{Q}{\sigma^2}\right)}{\exp\left(\frac{\alpha}{|y|}\frac{Q}{\sigma^2}\right) + \exp\left(-\frac{\alpha}{|y|}\frac{Q}{\sigma^2}\right)} \tag{27}$$

Again, this projection has the natural properties that $P(Q) \to (v+1-F)$ as $Q \to \infty$ and $P(Q) \to \max(0, v-1-F)$ as $Q \to -\infty$, and it converges more rapidly when α becomes larger (indicating a greater proportion of informed trade) and $|y|$ becomes smaller (indicating a smaller proportion of uninformed trade), and finally as σ^2 becomes smaller (indicating a smaller noise in the relative distribution of the two types of hedgers).

What we are primarily interested in is working out whether the optimal hedge demand satisfies (11) in Lemma 1. If it does not we can conclude that capital structure must play a non-trivial role as the hedgers' end of period wealth then becomes a function of F. Working out the difference between the asset value and the price projection in the two states, we find that

$$\begin{aligned} &\tilde{Y} - P(\tilde{Q}) \\ &= \begin{cases} \dfrac{(v+1-F)\exp\left(-\frac{\alpha}{|y|}\frac{Q}{\sigma^2}\right) - \max(0, v-1-F)\exp\left(-\frac{\alpha}{|y|}\frac{Q}{\sigma^2}\right)}{\exp\left(\frac{\alpha}{|y|}\frac{Q}{\sigma^2}\right) + \exp\left(-\frac{\alpha}{|y|}\frac{Q}{\sigma^2}\right)} = \overline{\tilde{Y} - P(\tilde{Q})} & \text{if High State} \\ -\dfrac{(v+1-F)\exp\left(\frac{\alpha}{|y|}\frac{Q}{\sigma^2}\right) - \max(0, v-1-F)\exp\left(\frac{\alpha}{|y|}\frac{Q}{\sigma^2}\right)}{\exp\left(\frac{\alpha}{|y|}\frac{Q}{\sigma^2}\right) + \exp\left(-\frac{\alpha}{|y|}\frac{Q}{\sigma^2}\right)} = \underline{\tilde{Y} - P(\tilde{Q})} & \text{if Low State} \end{cases} \end{aligned} \tag{28}$$

The expected utility of a type one hedger can, therefore, be written as the following expression, conditioning on the state of nature and using an arbitrary utility function u:

$$\begin{aligned} E(u_1(\tilde{w}_1)) = &\frac{1}{2}\left(\int_{-\infty}^{\infty} u\left(|y|\left(\overline{\tilde{Y} - P(\tilde{Q})}\right) - 1\right) f(Q - \alpha|y|)dQ\right) \\ &+\frac{1}{2}\left(\int_{-\infty}^{\infty} u\left(|y|\left(\underline{\tilde{Y} - P(\tilde{Q})}\right) + 1\right) f(Q + \alpha|y|)dQ\right) \end{aligned} \tag{29}$$

The marginal utility in $|y|$ can be therefore be worked out as

$$\begin{aligned} \frac{dE(u_1(\tilde{w}_1))}{d|y|} = &\frac{1}{2}\int_{-\infty}^{\infty} u'(\cdot)\left(\overline{\tilde{Y} - P(\tilde{Q})}\right) f(Q - \alpha|y|)dQ \\ &+\frac{1}{2}\int_{-\infty}^{\infty} u'(\cdot)\left(\underline{\tilde{Y} - P(\tilde{Q})}\right) f(Q + \alpha|y|)dQ \\ &+\frac{1}{2}\int_{-\infty}^{\infty} u'(\cdot)|y|\frac{d\left(\overline{\tilde{Y} - P(\tilde{Q})}\right)}{d|y|} f(Q - \alpha|y|)dQ \end{aligned}$$

$$+\int_{-\infty}^{\infty} u'(\cdot)|y|\frac{d\left(\tilde{Y}-P(\tilde{Q})\right)}{d|y|}f(Q+\alpha|y|)dQ$$
$$+\frac{1}{2}\int_{-\infty}^{\infty} u(\cdot)\frac{df(Q-\alpha|y|)}{d|y|}dQ+\frac{1}{2}\int_{-\infty}^{\infty} u(\cdot)\frac{df(Q+\alpha|y|)}{d|y|}dQ \tag{30}$$

The following result can be demonstrated.

Proposition 3 *Consider the smallest information cost which is prohibitive for information gathering and speculation, $c = 1$, which implies that $\alpha = 0$. Then we find that the derivative of the marginal utility with respect to α, evaluated at the point where $|y| = \frac{2}{(v+1-F)-\max(0,v-1-F)}$ and $\alpha = 0$,*

$$\frac{d}{d\alpha}\left(\frac{d}{d|y|}E(u(\tilde{w}))\right)\Bigg|_{|y|=\frac{2}{(v+1-F)-\max(0,v-1-F)},\alpha=0} \tag{31}$$

is strictly negative for sufficiently low values of σ^2 and strictly positive for sufficiently large values of σ^2.

The proof of this proposition, which is somewhat tedious given the complexity of the expression for marginal utility, can be found in the appendix. The implication of Proposition 3 is that capital structure decisions may have a non-trivial impact on the information gathering process but that the direction is unclear. We find, for instance, that if the parameter σ^2, which is the variance of the difference between the measure of type 1 and type 2 hedgers, is sufficiently small the inflow of speculators causes the hedgers to trade in smaller quantities than they otherwise would. Therefore, as speculators enter the market the hedgers would leave some risk uninsured which leads to a welfare loss. However, and more surprisingly, when the parameter σ^2 is sufficiently large the reverse is true and the uninformed traders trade more. This means that in some circumstances the hedgers withdraw trading orders as the informed traders enter, and in others they trade more. It is possible, therefore, that the hedgers increase their noise trading activity following an influx of informed trading in the market. The market may become more noisy and the prices less informative. Largely, however, a full analytical outline of these effects seems intractable.

It is, therefore, not possible to predict in the context of this more general model (at least for small values of α) which direction the information effects go. Increasing the information sensitivity of the security issued does not imply that the market prices become correspondingly more revealing. This finding has implications for the existing informational leverage literature, which makes general predictions about capital structure choice on the basis of informational effects alone. We argue that we need to know more about the modeling framework in order to arrive to the optimal borrowing policy for the firm. The elegant point of the informational leverage literature is that predictions can be made without such references but there is a danger they are too closely linked to the specific modeling framework applied.

Proposition 3 is related to a string of results looking at welfare effects caused by financial innovation. Two papers in particular are relevant in this context, these are Marin and Rahi (1999, 2000), both of which analyze in some detail the effects on welfare from information revelation. Factors that increase information revelation can serve as protection against adverse selection costs for the uninformed traders. This is an effect that is well known in the market microstructure framework, as it is the primary driving force behind the market maker's price setting policy. Factors that reduce information revelation can enhance risk sharing as hedging can take place before risk is revealed in prices. The latter effect is labeled the "Hirshleifer effect". Marin and Rahi (1999, 2000) explain this effect in the context of an Edgeworth box economy, where two symmetrically informed agents can always trade to a point on the contract curve as long as uncertainty is not revealed. Once uncertainty is revealed trading cannot take place and the agents' allocations may not be on the contract curve. Information revelation has, therefore, a direct negative impact on the welfare of the uninformed traders. These effects are by default ignored in models based on the Kyle-framework where noise traders trade for exogenous reasons. Marin and Rahi (1999) argue, for instance, that when a security becomes sensitive to speculative risk (i.e., sunspot risk or risk that is unrelated to fundamental risk in the economy), risk sharing can be enhanced, and this is more likely to happen in noisy markets. This effect also drives our result in that the hedgers benefit from the information revelation process generated by a small influx of informed traders when their trading activity is already quite noisy.

6 Conclusions

This paper analyzes informational leverage effect when noise traders behave rationally. In a simple, tractable, setting with endogenous noise trade we find that this effect disappears completely. However, informational leverage effects can emerge in generalizations of this model, but will now critically depend on assumptions about the way in which the endogenous noise trading activity is formed. The main message of the paper is, therefore, that modeling assumptions regarding noise traders are crucially important when drawing conclusions about information revelation effects in microstructure models of asset markets.

Appendix: Proofs

Proof of Lemma 1 First we assume that trading quantities are symmetric—i.e., the traders trade the same quantity when they buy as when they sell— which implies $y_1 = y = -y_2$ and $x \in \{-y, y\}$ for some number y to be determined later in the proof. Suppose $\alpha = 0$. The aggregate demand is $\tilde{\beta} y - (1 - \tilde{\beta})y$, which is uniform on $[-\xi y, \xi y]$ with density function $f(Q) = (2\xi y)^{-1}$ for $Q \in [-\xi y, \xi y]$ and $f(Q) = 0$ otherwise. Now allow $\alpha > 0$. If the speculators buy the asset, the aggregate demand is $Q = \alpha y + \tilde{\beta} y - (1 - \tilde{\beta})y$ which is uniform on $[-\xi y + \alpha y, \xi y + \alpha y]$, which has density function $f(Q - \alpha y)$. Similarly, if the speculators sell, the aggregate demand has density function $f(Q + \alpha y)$. Therefore, the revised beliefs of the market maker, contingent on making inferences from the aggregate demand Q only (as a pooling trading strategy is assumed), are given by

$$\Pr(\text{Asset value is high}|Q) = \frac{f(Q-\alpha y)}{f(Q-\alpha y)+f(Q+\alpha y)} = \begin{cases} 1 & \text{if } Q > \xi y - \alpha y \\ \frac{1}{2} & \text{if } -\xi y + \alpha y \le Q \le \xi y - \alpha y \\ 0 & \text{if } Q < -\xi y + \alpha y \end{cases} \tag{A.1}$$

This implies the price setting rule for the market maker. Next we need to confirm the optimal value of y. We make the observation that given the symmetry assumed, the expected utility for a hedger buying is the same as the expected utility for a hedger that is selling. We can, therefore, work on the expression for any given hedger so consider a type 1 hedger:

$$\begin{aligned} E[u(\tilde{w}_1(y))] = \frac{1}{2}\Bigg(&\int\limits_{y(\tilde{Y}-\tilde{P})\le 1} (y(\tilde{Y}-\tilde{P})-1)f(Q-\alpha y)dQ \\ &+ \int\limits_{y(\tilde{Y}-\tilde{P})>1} \gamma(y(\tilde{Y}-\tilde{P})-1)f(Q-\alpha y)dQ \Bigg) \\ +\frac{1}{2}\Bigg(&\int\limits_{y(\tilde{Y}-\tilde{P})\le -1} (y(\tilde{Y}-\tilde{P})+1)f(Q+\alpha y)dQ \\ &+ \int\limits_{y(\tilde{Y}-\tilde{P})>-1} \gamma(y(\tilde{Y}-\tilde{P})+1)f(Q+\alpha y)dQ \Bigg) \end{aligned} \tag{A.2}$$

First suppose $y \le \frac{2}{(v+1-F)-\max(0,v-1-F)}$. Then we find (using the probabilities $\Pr(\text{Asset value is high}|Q)$ derived above and the assertion that $\alpha < \xi$) that if $-\xi y + \alpha y \le Q \le \xi y - \alpha y$ there is no information revelation in prices, so that $\tilde{Y} - \tilde{P}$ equals $\frac{(v+1-F)-\max(0,v-1-F)}{2}$ over this region, conditional on a high asset value. Similarly, $\tilde{Y} - \tilde{P}$ equals $-\frac{(v+1-F)-\max(0,v-1-F)}{2}$ over the same region conditional on a low asset value. If $Q > \xi y - \alpha y$ (this happens only if the asset value is high) there is full information revelation, so that $\tilde{Y} - \tilde{P} = 0$. If $Q < -\xi y + \alpha y$ (this happens only if the asset value is low) there is also full information revelation, so that $\tilde{Y} - \tilde{P} = 0$ also over this region. Thus, we can rewrite the expected utility as

$$
\begin{aligned}
&E[u(\tilde{w}_1(y))] \\
&= -\frac{1}{2}\left(\int_{-\xi y+\alpha y}^{\xi y-\alpha y}\left(y\frac{(v+1-F)-\max(0,v-1-F)}{2}-1\right)\frac{1}{2\xi y}dQ\right. \\
&\qquad\left.+\int_{-\xi y-\alpha y}^{-\xi y+\alpha y}\frac{1}{2\xi y}dQ\right) \\
&\quad+\frac{\gamma}{2}\left(\int_{-\xi y+\alpha y}^{\xi y-\alpha y}\left(y\frac{(v+1-F)-\max(0,v-1-F)}{2}+1\right)\frac{1}{2\xi y}dQ\right. \\
&\qquad\left.+\int_{\xi y-\alpha y}^{\xi y+\alpha y}\frac{1}{2\xi y}dQ\right)
\end{aligned}
\tag{A.3}
$$

The expected marginal utility is, therefore, positive in y as long as y is in the region assumed and $\gamma < 1$. Now, suppose $y > \frac{2}{(v+1-F)-\max(0,v-1-F)}$. In this case the expected utility is precisely the negative of the expression above, so in this case the expected marginal utility is negative in y. The optimal trading strategy for a hedger of type 1 is, therefore,

$$
y = \frac{2}{(v+1-F)-\max(0,v-1-F)} \tag{A.4}
$$

The result follows. ||

Proof of Lemma 2 The speculators make trading profits only if the price is not fully revealing. The speculators make the same trading profits if the information is positive as if the information is negative. Suppose the speculators have positive information, in which case the price is fully revealing when $Q > \frac{2\xi}{(v+1-F)-\max(0,v-1-F)}$. We find, therefore, (using the fact that $\beta \sim U\left[\frac{1}{2}-\frac{\xi}{2},\frac{1}{2}+\frac{\xi}{2}\right]$)

$$
\begin{aligned}
&\Pr\left(Q \le \frac{2\xi}{(v+1-F)-\max(0,v-1-F)}\right) \\
&= \Pr\left(\beta y_1 + (1-\beta)y_2 \le \frac{2\xi}{(v+1-F)-\max(0,v-1-F)} - \alpha x\right) \\
&= \Pr\left(-\beta + (1-\beta) \le \xi - \alpha\right) \\
&= \Pr\left(\beta \ge \frac{1-\xi+\alpha}{2}\right) \\
&= 1 - \frac{\alpha}{2\xi}
\end{aligned}
\tag{A.5}
$$

Consequently, the probability the price is not fully revealing is $\frac{2\xi-\alpha}{2\xi}$. We find, therefore, that the speculators earn the following expected profit ex post:

$$\begin{aligned}
\text{Trading profit} &= \frac{2\xi-\alpha}{2\xi}\left((v+1-F)-((v+1-F)+\max(0, v-1-F))\frac{1}{2}\right) \\
&\quad \times \frac{2}{(v+1-1)-\max(0, v-1-F)} \\
&= \frac{2\xi-\alpha}{2\xi}
\end{aligned} \tag{A.6}$$

Since the speculators are competing perfectly in the market for information, the trading profits equal the information cost c. Consequently,

$$\alpha^{*} = 2\xi(1-c) \tag{A.7}$$

We obtain a similar argument assuming the speculators have negative information. Substitution of α^{*} back into the expression for $\Pr\left(Q \leq \frac{2\xi}{(v+1-F)-\max(0,v-1-F)}\right)$ yields the rest of the result. ||

Proof of Proposition 1 Lemmas 1 and 2 imply the basic result. We only need to verify that the hedgers' welfare is independent of the capital structure decision, which is demonstrated in the main text, equation (14). ||

Proof of Proposition 3 Throughout, we assume that $|y| = \frac{2}{(v+1-F)-\max(0,v-1-F)}$ and investigate the sign of the first order condition for a hedger of type 1 (we get a similar expression for a hedger of type 2). If negative, the hedger is trading quantities that are too small to be optimal, and if positive, the hedger is trading quantities that are too large. We apply this simple logic as a basis for proving the result. Before we proceed, however, it is useful to rewrite some of the expressions. Using the expressions for $\tilde{Y} - P(\tilde{Q})$ in (24), we find that

$$|y|(\overline{\tilde{Y} - P(\tilde{Q})}) - 1 = \frac{2}{1+\exp(2\alpha Q/|y|\sigma^{2})} - 1 \tag{A.8.1}$$

and that

$$|y|(\underline{\tilde{Y} - P(\tilde{Q})}) + 1 = -\frac{2}{1+\exp(-2\alpha Q/|y|\sigma^{2})} + 1 \tag{A.8.2}$$

Both expressions are negative for $Q \geq 0$ and positive for $Q < 0$. We can, therefore, define an operator $g(Q, x)$ which equals x for $Q \geq 0$ and γx for $Q < 0$. Thus, all terms in (30) involving the marginal utility u' can be written as $g(Q, 1)$ and all terms involving utility $u(x)$ can be written as $g(Q, x)$. We note that g is linear in the second argument so $g(Q, x) = g(Q, 1)x$. Next, we make the observation that

$$|y|\overline{(\tilde{Y} - P(\tilde{Q}))} - 1 = \frac{\exp(-\alpha Q/|y|\sigma^2) - \exp(\alpha Q/|y|\sigma^2)}{\exp(-\alpha Q/|y|\sigma^2) + \exp(\alpha Q/|y|\sigma^2)} = |y|\underline{(\tilde{Y} - P(\tilde{Q}))} + 1 \tag{A.9}$$

and that the following derivatives can be written as

$$\frac{d}{d|y|} f(Q - \alpha|y|) = \left(-\frac{1}{|y|} + \frac{Q(Q - \alpha|y|)}{|y|^3\sigma^2}\right) f(Q - \alpha|y|) \tag{A.10.1}$$

$$\frac{d}{d|y|} f(Q + \alpha|y|) = \left(-\frac{1}{|y|} + \frac{Q(Q + \alpha|y|)}{|y|^3\sigma^2}\right) f(Q + \alpha|y|) \tag{A.10.2}$$

$$\frac{d}{d|y|}\overline{(\tilde{Y} - P(\tilde{Q}))} = \frac{2\alpha Q}{|y|^2\sigma^2} \frac{\exp(2\alpha Q/|y|\sigma^2)}{1 + \exp(2\alpha Q/|y|\sigma^2)}\overline{(\tilde{Y} - P(\tilde{Q}))} \tag{A.10.3}$$

$$\frac{d}{d|y|}\underline{(\tilde{Y} - P(\tilde{Q}))} = -\frac{2\alpha Q}{|y|^2\sigma^2} \frac{\exp(-2\alpha Q/|y|\sigma^2)}{1 + \exp(-2\alpha Q/|y|\sigma^2)}\underline{(\tilde{Y} - P(\tilde{Q}))} \tag{A.10.4}$$

Combining all this information, we write the expression for marginal utility in (30) as (we drop the subscript denoting type 1 hedgers):

$$\begin{aligned}
&\frac{d}{d|y|}E(u(\tilde{w}))\\
&= \frac{1}{2}\int_{-\infty}^{\infty} g\left(Q, \frac{Q(Q-\alpha|y|)}{|y|^3\sigma^2}\frac{\exp(-\alpha Q/|y|\sigma^2)-\exp(\alpha Q/|y|\sigma^2)}{\exp(-\alpha Q/|y|\sigma^2)+\exp(\alpha Q/|y|\sigma^2)} f(Q-\alpha|y|)\right) dQ\\
&\quad + \frac{1}{2}\int_{-\infty}^{\infty} g\left(Q, \frac{Q(Q+\alpha|y|)}{|y|^3\sigma^2}\frac{\exp(-\alpha Q/|y|\sigma^2)-\exp(\alpha Q/|y|\sigma^2)}{\exp(-\alpha Q/|y|\sigma^2)+\exp(\alpha Q/|y|\sigma^2)} f(Q+\alpha|y|)\right) dQ\\
&\quad + \frac{1}{2}\int_{-\infty}^{\infty} g\left(Q, \frac{1}{|y|} f(Q-\alpha|y|)\right) dQ - \frac{1}{2}\int_{-\infty}^{\infty} g\left(Q, \frac{1}{|y|} f(Q+\alpha|y|)\right) dQ\\
&\quad + \frac{1}{2}\int_{-\infty}^{\infty} g\left(Q, \frac{\alpha Q}{|y|^2\sigma^2}\left(\frac{2}{\exp(\alpha Q/|y|\sigma^2)+\exp(-\alpha Q/|y|\sigma^2)}\right)^2 f(Q-\alpha|y|)\right) dQ\\
&\quad - \frac{1}{2}\int_{-\infty}^{\infty} g\left(Q, \frac{\alpha Q}{|y|^2\sigma^2}\left(\frac{2}{\exp(\alpha Q/|y|\sigma^2)+\exp(-\alpha Q/|y|\sigma^2)}\right)^2 f(Q+\alpha|y|)\right) dQ
\end{aligned} \tag{A.11}$$

It is useful to rewrite the marginal utility above in terms of the common density function $f(Q)$ which enables us to write integrals between zero and infinity. We also collect some of the exponential terms using the hyperbolic sine and cosine functions. We thus obtain:

$$\begin{aligned}
&\frac{d}{d|y|}E(u(\tilde{w}))\\
&= -\int_0^\infty \frac{(1-\gamma)Q^2}{|y|^3\sigma^2}\exp\left(-\frac{\alpha^2}{2\sigma^2}\right)\sinh\left(\frac{\alpha Q}{|y|\sigma^2}\right)f(Q)dQ\\
&\quad+\int_0^\infty \frac{(1-\gamma)\alpha Q}{|y|^2\sigma^2}\exp\left(-\frac{\alpha^2}{2\sigma^2}\right)\sinh^2\left(\frac{\alpha Q}{|y|\sigma^2}\right)\cosh^{-1}\left(\frac{\alpha Q}{|y|\sigma^2}\right)f(Q)dQ\\
&\quad+\int_0^\infty \frac{1-\gamma}{|y|}\exp\left(-\frac{\alpha^2}{2\sigma^2}\right)\sinh\left(\frac{\alpha Q}{|y|\sigma^2}\right)f(Q)dQ\\
&\quad+\int_0^\infty \frac{(1+\gamma)\alpha Q}{|y|^2\sigma^2}\exp\left(-\frac{\alpha^2}{2\sigma^2}\right)\sinh\left(\frac{\alpha Q}{|y|\sigma^2}\right)\cosh^{-2}\left(\frac{\alpha Q}{|y|\sigma^2}\right)f(Q)dQ
\end{aligned} \tag{A.12}$$

Now turn to the problem of the speculators. Their expected trading profits equals

$$\frac{1}{2}E|y|\overline{(\tilde{Y}-P(\tilde{Q}))}-\frac{1}{2}E|y|\underline{(\tilde{Y}-P(\tilde{Q}))} \tag{A.13}$$

which, with some algebra, can be written as

$$\begin{aligned}
&\int_{-\infty}^{\infty}\frac{2\exp(-\alpha^2/2\sigma^2)}{\exp(\alpha Q/|y|\sigma^2)+\exp(-\alpha Q/|y|\sigma^2)}f(Q)dQ\\
&=\exp(-\alpha^2/\sigma^2)\int_{-\infty}^{\infty}\cosh^{-1}(\alpha Q/|y|\sigma^2)f(Q)dQ
\end{aligned} \tag{A.14}$$

Since the cosh function is always greater than or equal to 1, the expectation of its inverse is always less than or equal to one. At the point where $\alpha=0$, the cosh function equals exactly 1, so the expected trading profits equal 1. An information cost of $c=1$ is the smallest cost that prevents informed speculation in the market, therefore. Considering the marginal utility of the hedgers at this point, we find that their first order condition (subject to the constraint on $|y|$) of utility maximization can be written as

$$
\begin{aligned}
0 = &-\int_0^\infty \frac{1-\gamma}{|y|}\left(\left(\frac{Q}{|y|\sigma}\right)^2 - 1\right)\sinh\left(\frac{\alpha Q}{|y|\sigma^2}\right) f(Q)dQ \\
&+\int_0^\infty \frac{\alpha Q}{|y|^2\sigma^2}\sinh\left(\frac{\alpha Q}{|y|\sigma^2}\right)\cosh^{-1}\left(\frac{\alpha Q}{|y|\sigma}\right) \\
&\times\left((1-\gamma)\sinh\left(\frac{\alpha Q}{|y|\sigma^2}\right) + (1+\gamma)\cosh^{-1}\left(\frac{\alpha Q}{|y|\sigma^2}\right)\right) f(Q)dQ \qquad \text{(A.15)}
\end{aligned}
$$

which clearly is satisfied due to the fact that the terms involving $\sinh\left(\frac{\alpha Q}{|y|\sigma^2}\right)$ are zero at the point where $\alpha = 0$.

Now consider the cross derivative $\frac{d}{d\alpha}\left(\frac{d}{d|y|}E(u(\tilde{w}))\right)$ at the point where $\alpha = 0$ and $|y|$ is given by Eq. (11) in the main text:

$$
\begin{aligned}
&\left.\frac{d}{d\alpha}\left(\frac{d}{d|y|}E(u(\tilde{w}))\right)\right|_{\alpha=0} \\
&= -\int_0^\infty \frac{1-\gamma}{|y|}\left(\left(\frac{Q}{|y|\sigma}\right)^2 - 1\right)\frac{d}{d\alpha}\sinh\left(\frac{\alpha Q}{|y|\sigma^2}\right) f(Q)dQ \\
&= -\int_0^\infty \frac{1-\gamma}{|y|}\left(\left(\frac{Q}{|y|\sigma}\right)^2 - 1\right)\frac{Q}{|y|\sigma^2} f(Q)dQ \qquad \text{(A.16)}
\end{aligned}
$$

Ignoring constants, the sign of the right hand side is determined by

$$
\int_0^\infty Q^2 f(Q)dQ - \frac{1}{|y|^2\sigma^2}\int_0^\infty Q^3 f(Q)dQ \qquad \text{(A.17)}
$$

which can be simplified, applying integration by parts to the second term, to the expression

$$
\int_0^\infty \left(Q^2 - \frac{2}{|y|^2\sigma^2}Q\right) f(Q)dQ = \int_0^\infty Q\left(Q - \frac{2}{|y|^2\sigma^2}\right) f(Q)dQ \qquad \text{(A.18)}
$$

If we let $\sigma^2 \to \infty$, this expression approaches $\int_0^\infty Q^2 f(Q)dQ = \frac{1}{2}\mathrm{Var}(Q)$ which is always positive. If we let $\sigma^2 \to 0$, the expression approaches $-\infty$ which is negative. There exists, therefore, a value $\sigma \in (0, \infty)$ for which the expression is zero and the first order condition remains satisfied for small changes in α. ||

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